

PSCMTH18 - Integral Equations

P. Pages : 2

Time : Three Hours

**GUG/S/25/13769**

Max. Marks : 100

- Notes : 1. Solve all the questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Show that _____ is a solution of the Volterra integral equation **10**

$$u(x) = \sin x + 2 \int_0^x \cos(x-t) u(t) dt$$
- b) Obtain the integral equation from **10**

$$\frac{d^3 y}{dx^3} + x \frac{d^2 y}{dx^2} + (x^2 - x)y = xe^x + 1, \text{ with } y(0) = 1, y'(0) = 1, y''(0) = 0$$

OR

- c) Find the Fredholm integral equation of second kind to the boundary value problems **10**

$$\frac{d^2 u}{dx^2} + \lambda u = x, \text{ with } u(0) = 0 \text{ \& } u'(1) = 0.$$
- d) Form the integral equation for the DE, $y''' - 2xy = 0$ with $y(0) = \frac{1}{2}, y'(0) = 1 \text{ \& } y''(0) = 1.$ **10**

UNIT – II

2. a) Find the eigen values and eigen functions of the homogeneous integral equation **10**

$$u(x) = \lambda \int_0^1 e^x e^t u(t) dt$$
- b) Show that the homogeneous integral equation $u(x) = \lambda \int_0^1 (t\sqrt{x} - x\sqrt{t}) u(t) dt$ does not have real eigen values & eigen functions. **10**

OR

- c) Solve : $u(x) = 1 + \int_0^1 (1 + e^{x+t}) u(t) dt$ **10**
- d) Solve : $u(x) = f(x) + \lambda \int_{-1}^1 (xt + x^2 t^2) u(t) dt$ **10**

UNIT – III

3. a) Show that the eigen functions of a symmetric kernel corresponding to different eigen values are orthogonal. **10**

- b) Solve the homogeneous Fredholm integral equation by Schmidt theorem. 10

$$f(x) = \lambda \int_0^1 e^x e^t f(t) dt$$

OR

- c) Solve the symmetric integral equation $u(x) = f(x) + \lambda \int_a^b k(x) k(t) u(t) dt$ 10
- d) Solve : $u(x) = e^x + \lambda \int_0^1 (5x^2 - 3)t^2 u(t) dt$ 10

UNIT – IV

4. a) Find the iterated Kernel of $k(x, t) = e^x \cos t$, $a = 0, b = \pi$ 10
- b) Solve the Fredholm integral equation of the second kind $u(x) = 2x + \lambda \int_0^1 (x+t) u(t) dt$ by the method of successive approximation with $u_0(x) = 1$. 10

OR

- c) Solve : $u(x) = f(x) + \frac{1}{2} \int_0^1 e^{x-t} u(t) dt$ 10
- d) Solve the Volterra integral equation $u(x) = 1 + x + \lambda \int_0^x (x-t) u(t) dt$ 10
5. a) Define a Fredholm integral equation. 5
- b) Define the separable kernel. 5
- c) State the Hilbert-Schmidt theorem. 5
- d) When the two functions are said to be reciprocal? 5
